

2025秋, 有限元方法II, 作业1

交作业时间: 2025/09/25

The Mathematical Theory of Finite Element Methods:

- Chapter 0: 0.x.6, 0.x.11, 0.x.12, 0.x.13
- Chapter 1: 1.x.13, 1.x.26, 1.x.42

Supplementary Questions:

1. For any $0 < s < 1$, show that

$$\int_{\mathbb{R}^n} \frac{|e^{i\xi \cdot \eta} - 1|^2}{|\eta|^{n+2s}} d\eta = \frac{1}{c(n, s)} |\xi|^{2s},$$

where the positive constant is given by

$$c(n, s) = \frac{2^{2s} s \Gamma(s + n/2)}{2\pi^{n/2} \Gamma(1 - s)}.$$

2. Let $\Omega \subset \mathbb{R}^n$ be a bounded Lipschitz domain. Show that for any $q \geq 1$,

$$\|v\|_{L^q(\Omega)} \leq C(n) q^{1-\frac{1}{n}} |\Omega|^{\frac{1}{q}} \|v\|_{W^{1,n}(\Omega)} \quad \forall v \in W_0^{1,n}(\Omega).$$