

The 5th International Undergraduate Mathematics Summer School

Information Booklet

I. Brief Introduction

Dates:

Jul 28-Aug 8, 2025

Venue:

Room 202, Second Teaching Building of Peking University

Courses:

1. Shane Kelly (University of Tokyo)

Subject: K-theory and arithmetic geometry

2. Changkeun Oh (Seoul National University)

Subject: Introduction to decoupling theory

3. Gregory Chechkin (Lomonosov Moscow State University)

Subject: Shock Waves and Nonlinear First-Order Equations

4. Alexei Penskoi (Lomonosov Moscow State University)

Subject: Spectral Geometry

5. Weijun Xu (Peking University)

Subject: On Malliavin's proof of Hormander's theorem, and beyond

One-hour Lectures:

1. Keiji Oguiso (University of Tokyo)

Subject: Finiteness and infiniteness of real forms of a projective variety in the view of birational geometry and algebraic dynamics

2. Dohyeong Kim (Seoul National University)

Subject: Units in algebraic number fields

3. Alexei Davydov (Lomonosov Moscow State University and NUST MISIS)

Subject: Classification of linear second order partial differential equations on the plane

4. Yuriy Tarannikov (Lomonosov Moscow State University)

Subject: On Low-Degree Real Polynomials of Boolean Functions, Their Relevant Variables, and Related Topics

5. Wei Wang (Peking University)

Subject: Index iteration theory for Symplectic paths with applications to the closed geodesic problem

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Sponsor: School of Mathematical Sciences, Peking University

Partner University: Moscow State University, Seoul National University, University of Tokyo

II. Time Schedule:

Jul 28, 8:30-9:00 register

Date: 7.28-8.1

	Mon	Tue	Wed	Thu	Fri
9:10-10:00	Opening session and group photo	Penskoi	Penskoi	Penskoi	Penskoi
10:00-11:00	Tarannikov				
13:00-14:50	Oh	Oh	Oh	Oh	Oh
15:10-17:00	Penskoi	Davydov	Kim	Kelly	Chechkin
		Wang	Oguiso		

7.28 17:20 Visit to Zihua Building (School of Mathematical Sciences, PKU) and Beijing International Center for Mathematical Research (Peking University).

Date: 8.4-8.8

	Mon	Tue	Wed	Thu	Fri
9:10-11:00	Penskoi	Chechkin	Chechkin	Chechkin	Chechkin
13:00-14:50	Xu	Xu	Xu	Xu	Xu
15:10-17:00	Kelly	Kelly	Kelly	Kelly	

Support: Admitted students will be provided with campus dining cards, and admitted students from other cities will be provided with accommodation.

III. Introduction to courses and lectures

Courses

1. Shane Kelly (University of Tokyo)

Subject: K-theory and arithmetic geometry

Abstract: Algebraic K-theory was defined in the late 1950s by Alexander Grothendieck in order to formulate his generalisation of the Riemann–Roch theorem, which relates vector bundles on algebraic varieties to algebraic cycles. The foundational idea is to construct a group in which every short exact sequence of vector bundles splits. Since then, algebraic K-

theory has developed into a deep and far-reaching subject with applications across algebraic geometry, number theory, and topology.

For example, algebraic K-theory plays a central role in the study of regulators and the interpretation of special values of L-functions, with deep connections to the class number formula, the Birch and Swinnerton-Dyer conjecture, and other conjectures in arithmetic geometry such as those of Beilinson and Bloch–Kato.

In these lectures we will present a survey of some classical topics in K-theory, starting with K_0 as defined by Grothendieck and continuing to the higher K-theory defined by Quillen. If there is time, in the last lecture we may survey some open conjectures, recent advances, and currently active areas of research.

2. Changkeun Oh (Seoul National University)

Subject: Introduction to decoupling theory

Abstract: In 2014, a decoupling inequality was proved by Bourgain and Demeter. Since their work, many different proofs of the decoupling inequality for the parabola have been found. Among them, the proof using the high-low method is particularly notable because it provides the current best bound for the decoupling constant for the parabola. In this series of lectures, I will give a proof of a decoupling inequality for the parabola using a high-low method.

3. Gregory Chechkin (Lomonosov Moscow State University)

Subject: Shock Waves and Nonlinear First-Order Equations

Abstract: The course deals with the Hopf and Burgers equations, as well as nonlinear first-order equations. The local classical theory and weak solutions with shock waves in rarefaction waves are studied. Various examples are given and the Riemann problem of discontinuity decay is studied.

4. Alexei Penskoï (Lomonosov Moscow State University)

Subject: Spectral Geometry

Abstract: Spectral Geometry is a vibrant and rapidly evolving area of mathematics at the intersection of analysis, partial differential equations, topology, and differential geometry. It investigates how geometric properties of Euclidean domains and Riemannian manifolds are related to the eigenvalues and eigenfunctions of the Laplace operator defined on them.

Part I. Hear the Shape, See the Sound.

Prerequisites: Multivariable Calculus, Linear Algebra.

- 1) Introduction to the wave equation and the spectral problem for the Laplace operator with Dirichlet, Neumann, and other boundary conditions. Variational characterization of eigenvalues and basic inequalities.
- 2) Can one hear the shape of a drum? Isospectrality. Weyl's law and the asymptotic distribution of eigenvalues.

3) See the sound. Nodal lines and nodal domains of eigenfunctions. Nodal Geometry and Nodal Topology

Part II. How Large or Small Can an Eigenvalue Be?

Prerequisites: Multivariable Calculus, Linear Algebra, Differential Geometry.

4) Lord Rayleigh's question: Can one find a drum with the lowest fundamental frequency among drums of the same fixed area?. Faber-Krahn inequality. Critical metrics for eigenvalues on Riemannian manifolds.

5) The relationship between critical metrics and minimal submanifolds & harmonic maps.

6) Isoperimetric inequalities for eigenvalues on compact surfaces without boundary.

5. Weijun Xu (Peking University)

Subject: On Malliavin's proof of Hormander's theorem, and beyond.

Abstract: Hormander's famous "sums of squares" theorem (1967) gave a clean criterion on when a second order differential operator is hypoelliptic. Since it has clear interpretation in terms of diffusion processes, a probabilistic proof of this result has been a long-standing goal. Ten years after Hormander's original analytic proof, Malliavin (1978) achieved this by developing a stochastic calculus of variations, now known as Malliavin calculus.

We will introduce main ideas of these developments and sketch the proof of Hormander's theorem. After that, we discuss its impact on other problems, with a particular focus on ergodicity of diffusion processes in infinite dimensions. A milestone in this direction is the Hairer-Mattingly theory on ergodicity for SPDEs with highly degenerate noises.

Prior knowledge in stochastic analysis is not necessary.

One-hour lectures

1. Keiji Oguiso (University of Tokyo)

Subject: Finiteness and infiniteness of real forms of a projective variety in the view of birational geometry and algebraic dynamics

Abstract: Real form problem is the problem to ask how many different ways, up to isomorphisms over the real number field, to describe a given complex variety in terms of equations of real coefficients.

In this talk, I would like to report some unexpected applications of modern technique of algebraic geometry blended with algebro-dynamical view to this classical problem, finiteness or infiniteness of real forms of a complex projective variety mainly based on my joint works with several authors. Through this talk, I also would like to introduce several concrete examples of projective varieties.

2. Dohyeong Kim (Seoul National University)

Subject: Units in algebraic number fields

Abstract: I will give a brief introduction to algebraic number fields and the groups of units in them. Their abstract structure as abelian groups are determined by the celebrated theorem named after Dirichlet. Nevertheless, their arithmetic nature remains mysterious and take parts in the development of modern mathematics. I will illustrate this by some conjectures and recent progresses on them made by undergraduate students.

3. Alexei Davydov (Lomonosov Moscow State University and NUST MISIS)

Subject: Classification of linear second order partial differential equations on the plane

Abstract: The Laplace and wave equations are well known as canonical forms of second-order linear partial differential equations on a plane near points of elliptic and hyperbolic types of such equations, respectively. These equations were proposed in the 18th century (D. Bernoulli, J. d'Alembert, L. Euler, P.S. Laplace) to study a number of natural processes of various natures. However, such equations, generally speaking, can have different types in different regions of space. In a typical smooth situation, such subregions are separated by a smoothly embedded curve, and when crossing it, the type of the equation changes. The systematic study of equations of mixed type was first started by F. Tricomi in his 1923 treatise.

The lecture will cover a complete local classification of typical smooth linear partial differential equations of mixed type on a plane, and related solved and unsolved mathematical problems will be discussed.

4. Yuriy Tarannikov (Lomonosov Moscow State University)

Subject: On Low-Degree Real Polynomials of Boolean Functions, Their Relevant Variables, and Related Topics

Abstract: This lecture will focus on bounds for the maximum number of relevant variables in Boolean functions represented by low-degree real polynomials. We begin with the Nisan-Szegedy bound, then discuss its refinements by Chiarelli-Hatami-Saks and Wellens. A construction of a function achieving the best-known lower bound for this maximum will be presented.

We will also explore connections between this problem and high-order resilient functions, demonstrating the complete equivalence between the problem of maximizing the number of nonlinear variables in such functions and the central question of this lecture. Along the way, key concepts like Walsh (Fourier) coefficients, block sensitivity, and hitting sets will be introduced, providing useful tools for the analysis.

5. Wei Wang (Peking University)

Subject: Index iteration theory for Symplectic paths with applications to the closed geodesic problem

Abstract: This lecture will introduce the index iteration theory for Symplectic paths developed by Y. Long and his coworkers. As an application of this theory, we use it to study the closed geodesic problems.

IV Accommodation and Dining

Foreign faculty and students can check into the FX hotel (富驿时尚酒店) on 27/07 and check out by 12:00 noon on 09/08.

Address: No.68 North 4th Ring West Road (South of Haidian Bridge) Haidian District, Beijing

Tel: +8610 58986688

The 富驿时尚 hotel is marked in the map, you can walk to school through the South Gate or the Southwest Gate. The Southwest Gate is the closest, and there is a canteen on the way to the classroom.



Dining

Foreign faculty and students will be provided with campus dining cards. You can have meal at JIAYUAN(家园)、YANNAN(燕南)、YIYUAN(艺园)、TONGYUAN(佟园清真餐)、CHANGCHUNYUAN(畅春园, 西门外) canteen and Campus food truck with dining cards.

Tips: YIYUAN is the nearest to the South-west Gate.

Instructions for Using the Physical Dining Card:

1. This card is non-registered. **If lost, it would not be reissued.** Please ensure proper safekeeping.
2. The PIN for card transactions is 011397. This may be required for authentication when expenditures reach specified limits.
3. **Non-refundable.** Ensure balance is fully depleted before departure, or gift the physical card to other event participants for continued use.

V Transportation

It is very convenient to take subway for your travel. **Subway line 4 has a stop at the east gate of Peking University.** It is connected with other subway lines, train stations and airport express railway. If you take the airport express railway at the Beijing Capital International Airport, please transfer to subway line 10 at the San-Yuan-Qiao stop, then transfer to line 4 at the Hai-Dian-Huang-Zhuang stop, and then get off at the East Gate of Peking University stop.

The nearest metro station to your accommodation hotel is **Wanquanhe Qiao Station on Line 16.**

Having more than 15 lines, Beijing subway could bring you to almost every corner of the city.



Peking University Campus Map

